Logic Appendix:

More detailed instruction in deductive logic

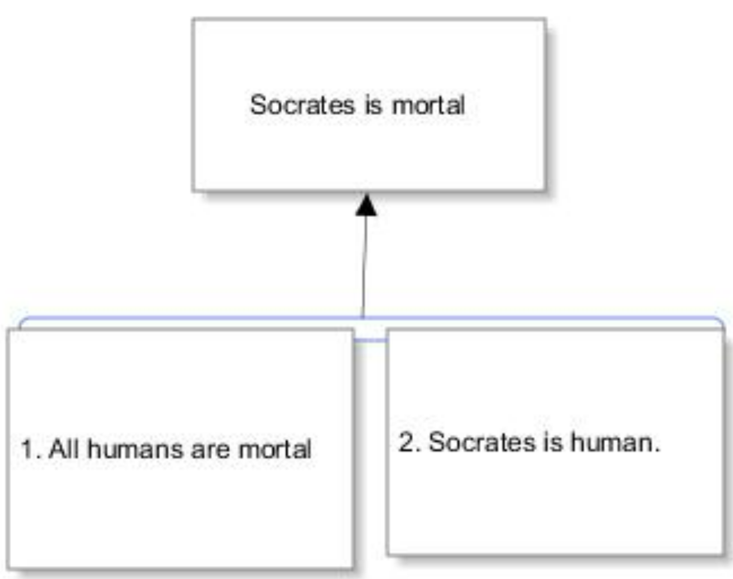
Standardizing and Diagramming

In *Reason and the Balance* we have taken the approach of using a simple outline to standardize short arguments, but we can also standardize arguments by creating diagrams of the argument. Such diagrams often help us to see which claims are premises supporting the conclusion and which are premises supporting other premises in a sub-argument. They also illustrate a fundamental distinction between the way the premises support their conclusion. Some premises are **linked** to other premises in order to support the conclusion. Other premises called **convergent** premises, each provide independent support for the conclusion. Let's look at few examples to see the difference. The famous argument about Socrates from Chapter 3:

1. All humans are mortal
2. Socrates is human

Conclusion Socrates is mortal

can be diagrammed as follows:



Intuitively we recognize that the two premises work together to support the conclusion, but we should also recognize that it is a characteristic of deductive arguments generally that the premises are linked.

The diagram shows the two premises working together to provide support for the conclusion. We can compare this argument to the following brief selection from the dialogue in Chapter 4 concerning the legalization of prostitution. In this statement Winnie is arguing against the claim that prostitution is immoral.

> ***Winnie:*** *What's morality got to do with it? Why would you think that an act between consenting adults and where there are no victims is immoral?*

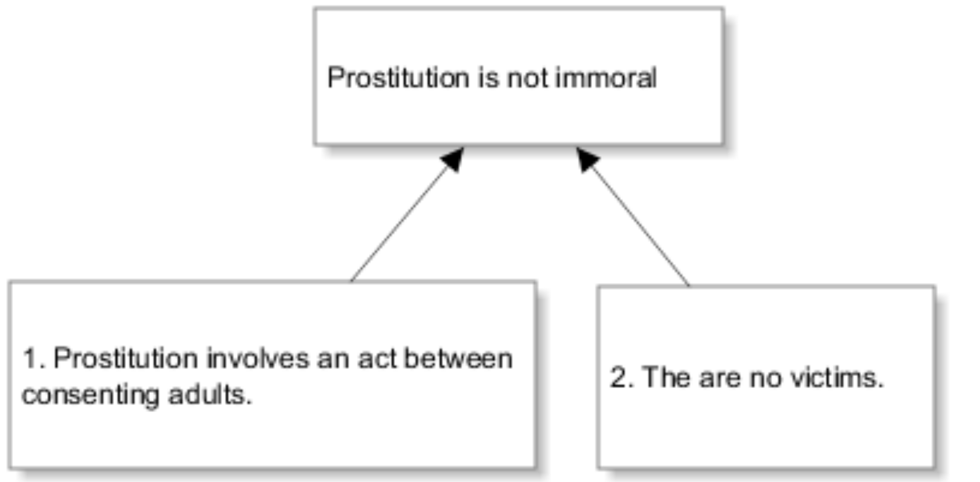
Standardizing the argument we would get:

1. Prostitution involves an act between consenting adults

2, There are no victims

Conclusion: Prostitution is not immoral

While standardizing typically puts the conclusion as the last line, when diagramming we often put the conclusion at the top of the diagram. This way of representing an argument illustrates the metaphor of support. The premises below "support" the conclusion in much the same way that a foundation supports a house. But when looking at deductive arguments, we often use the metaphor of "path"—the argument "leads' to the conclusion. Going from premises down to the conclusion would model this way of looking at arguments. Because most of the arguments we will be looking at in this text are non-deductive, we diagram arguments with the conclusion at the top.



Note that the two premises provide independent support -- not linked support. The difference is that each premise provides some support for the conclusion independently of the other. In the first argument about Socrates, the two premises were linked together to support the conclusion.

To see the significance of the two different kinds of argument support, let's take Nancy's argument in favor of prostitution being illegal:

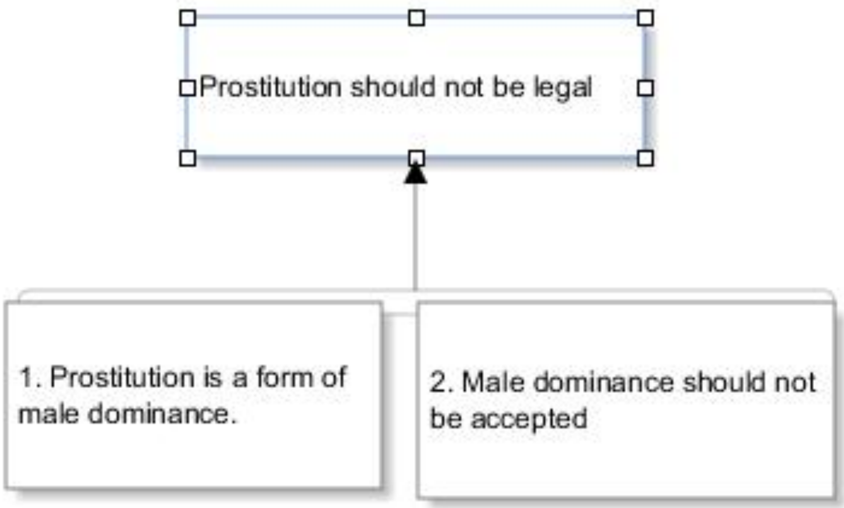
Nancy: *Wait just a minute, you two. You're both missing the point. Prostitution is demeaning to women. It's just another form of male dominance over women and should not be accepted.*

A possible standardization of the argument is:

1. Prostitution is a form of male dominance

2. Male dominance should not be accepted

Conclusion: Prostitution should not be legal



Here again the premises are linked because neither one provides any independent support for the conclusion but together they do provide some support. But this argument also demonstrates that because the premises are linked, if one premise is weak, the whole argument is weakened.

In linked arguments the premises do not provide direct, independent support for the conclusion, but work together like links of a chain. As with a chain, linked arguments are only as strong as the weakest link (premise). Showing that one linked premise is weak or false undermines the whole argument's support for its conclusion.

In a **convergent argument,** each premise independently provides support for the conclusion. Showing that any particular premise is weak, false or simply not credible does not necessarily weaken the total support for the conclusion. The metaphor here is one of support: a porch could be supported by numerous uprights and the fact that one is weak does not mean that the floor is not adequately supported by the others.

Knowing whether an argument is linked or convergent will help us considerably when we come to evaluate arguments.

We can also use diagrams to illustrate the role of sub-arguments:

Remember how we standardized McGregor's argument in chapter 3:

Conclusion: Minimum wage should not be raised (This replaces "is a dumb idea.")

1. Measures that put undue burdens on business are bad for the economy.

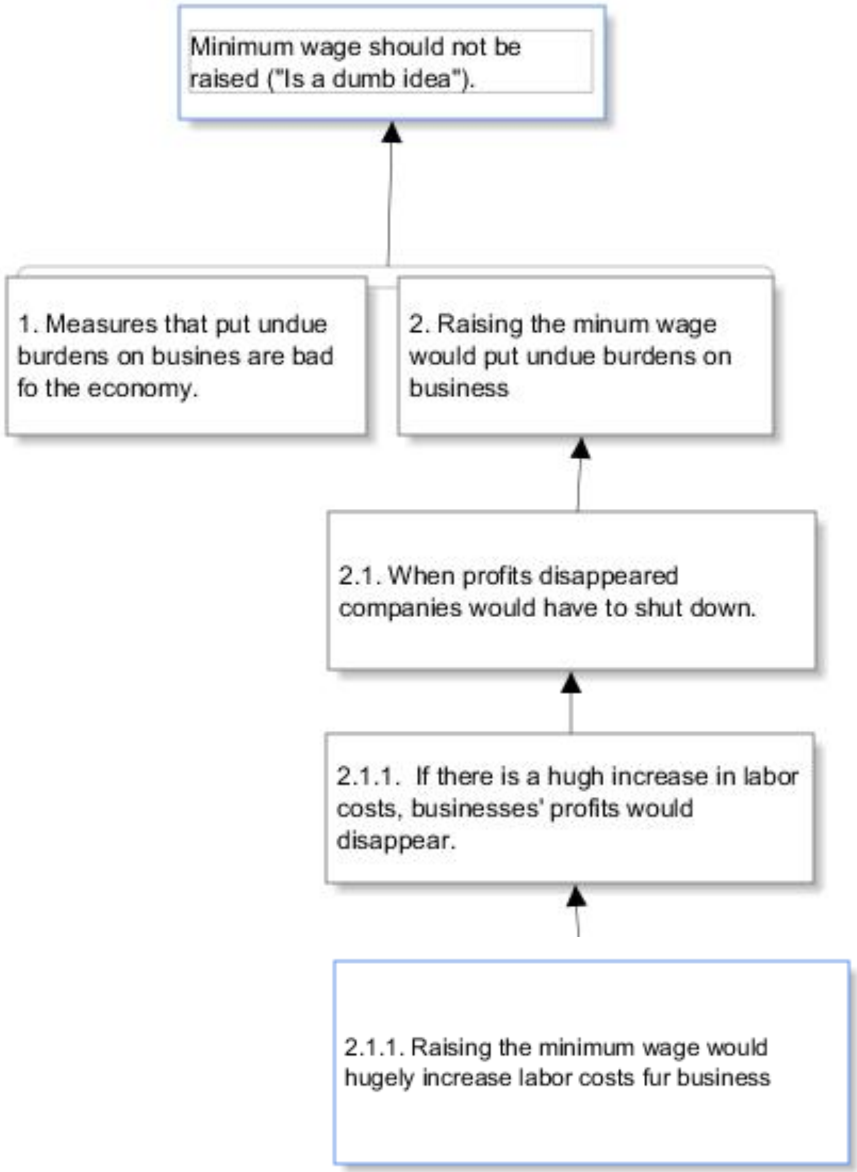
2. Raising the minimum wage would put undue burden on business.

2.1. When profits disappeared, companies would have to shut down.

2.1.1. If there's a huge increase in labor costs, businesses' profits would disappear.

2.1.1.1. Raising the minimum wage would hugely increase labor costs for businesses.

This can be diagrammed as follows:



This is just a brief introduction to diagramming. To deepen your understand and to practice diagramming **click here** for an excellent web-based introduction to diagramming from Carnegie Mellon. You can also **download ilogos** **--** a diagramming tool used in the Carnegie Mellon course and used to create the diagrams in this appendix.

Deduction and Categorical or Syllogistic Reasoning

We introduced the concepts of deduction and categorical reasoning in Chapter 3, and although we do not spend much time using this type of argument in the text, deductive arguments are often seen as the paradigm of arguments and have been extensively studied since the time of Aristotle. So what follows is a somewhat more extensive introduction to this form of reasoning.

While the study of deductive argument begins with Aristotle, the formalization of deductive arguments began in the late 19th century and was extensively developed in the 20th century. This new formalism is one of the intellectual tools that enabled the development of the digital computer.

An important aspect of deductive arguments is that their validity is based on the underlying form. As we know, a deductive argument is valid when, if the premises are true, the conclusion must be true. This entailment relationship in an argument is a result of the argument being an instance of a **valid argument form.**

Let us first look at the argument forms for categorical or syllogistic reasoning. Syllogistic reasoning involves claims (sentences, propositions) that assert or deny that a category (crows) is a member of some other category (birds) or that an individual (Socrates) is a member of a category (humans).

Abstracting from the famous Socrates argument

All humans are mortal

Socrates is human

Conclusion: Socrates is mortal

we can see that the underlying form of the argument is:

All A's are B

X is an A

Conclusion: X is a B

An argument form is a **valid argument form** when, if the variables in the argument form are filled in so that the premises are true, the conclusion must be true. By parity of reasoning, an argument form **is not a valid argument form** if the premises could be true, but the conclusion false.

Let's look at another argument that Nancy might have made in Chapter 3:

> ***McGregor**: Measures that are good for the economy will ultimately help the poor.*
>
> ***Nancy:** Measures that are good for the economy will help the poor? Well, raising the minimum wage will help the poor. So by your own reasoning, it must be good for the economy.*
>
> ***McGregor**: That doesn't sound quite right to me...*

Standardizing

1. Measures that are good for the economy will help the poor
2. Raising the minimum wage will help the poor

Conclusion: Raising the minimum wage must be good for the economy.

McGregor is justifiably reluctant to accept this reasoning. Why? Because this argument is not valid. Nancy's argument has the following form:

All A's are B	All measures that are good for the economy are measures that will help the poor
X is a B	Raising the minimum wage is a measure that will help the poor
Conclusion: X is an A	Raising the minimum is a measure that is good for the economy

To see why this form is not valid, look at the following instance of the form:

1. All rabbits are furry
2. My cat is furry

Conclusion My cat is a rabbit

The premises can be true, but the conclusion false. It is clearly not valid.

Do you think that the following argument is valid or invalid?

1. All Ravens are birds
2. All birds are green

Conclusion: All Ravens are green.

Think carefully. If the premises were true, would that make the conclusion true? Of course premise 2 is not true and neither is the conclusion, but that is <u>irrelevant to the question of validity</u>. The argument is valid! This odd argument should remind us of two key points about valid deductive arguments. 1. What makes the argument valid is the underlying argument form. Therefore the validity of the argument is independent of whether a particular substitution makes the premises true. So whether an argument is valid depends on whether it is an instance of a valid argument form, not whether its premises are true or its conclusion true.

Here is the form of the Raven argument above:

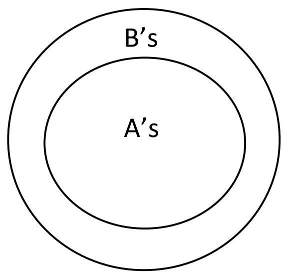
All A's are B

All B's are C's

Conclusion: All A's are C's.

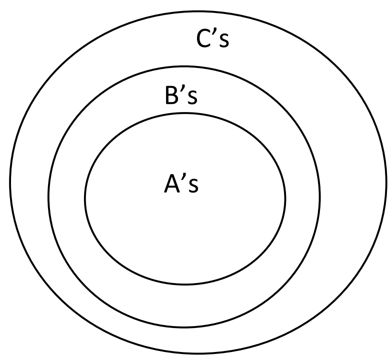
By using circles we can illustrate why this argument is valid.

1. All A's are B's



Now add a circle for the second premise

2. All B's are C's



We can clearly see that all the A's are inside all the C's

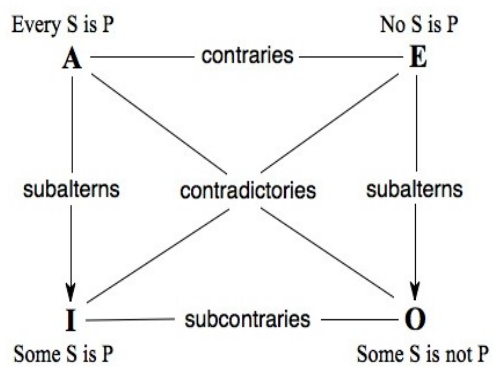
You might think of *valid argument form* as a calculator. As the saying goes, "Garbage in, garbage out." The *validity* of the calculator is not undermined if you type in the wrong numbers and then get a wrong answer. What the calculator guarantees is that *if* you type in the right numbers (and operation signs) you will get the right answer. The same is true of putting true claims into a valid argument form.

Before going on to study syllogistic arguments in more detail, we should note that there are four forms of categorical claims which have a traditional letter designation:

- Universal Affirmative (A-statement): All ravens are birds.
- Universal Negative (E-statement): No birds are mammals.
- Particular Affirmative (I-statement): Some senators are corrupt.
- Particular Negative (O-statement): Some birds cannot fly.

We can visualize their relationship in what is called the square of opposition:

Square of Opposition



The meaning of the terms in the square is explained below:

- **Contradictories**: One statement is true, and the other is false.
- **Contraries**: The two statements cannot be both true.
- **Subcontraries**: The two statements cannot be both false.
- **Subalterns**: If a **superaltern** is true (i.e., either of the generalizations on the first line), then the **subaltern** (bottom of square) is also true. (E.g., If all crows are black, then clearly some crows are black)

The most important aspect of the square of opposition to note is that when you want to deny (contradict) a universal claim like "All humans are selfish" you do not need to assert "No humans are selfish" but rather merely assert the contradictory "Some humans are not selfish". (See p. 64 of the text.) "Some" in this case means at least one. This is why anecdotal evidence

(“My friend Mary is a totally unselfish person”) can be used to refute (contradict) a generalization such as “No humans are selfish” despite the fact that such examples cannot support generalizations.

A more complex use of circles to illustrate all the validity and invalidity relationships of syllogistic arguments is done with Venn diagrams. **Click here for an excellent introduction** to the use of Venn diagrams. You can also **click here for some examples** that will test your understanding of Venn diagrams.

In the next section we will move from looking at categorical/syllogistic logic to a form of logic that is more easily applied to arguments in ordinary language: **propositional logic**. Studying it is more likely to help us with everyday reasoning.

Propositional Logic

A *proposition* is a statement which has a truth value: it is either true or false. Propositional logic[1] is the logic that is concerned with the rules for reasoning about propositions. Basically, there are two kinds of propositions: simple and compound. A proposition is compound if it is obtained from one or more propositions linked by propositional connectives.

By *propositional connectives*, we mean words which connect two propositions to form a new one. These are called *truth-functional connectives*. If a **compound proposition** is obtained from simple propositions connected by truth-functional connectives, then its truth value can be worked out based on the truth value(s) of its component propositions. There are four basic truth-functional connectives:

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[1] In some textbooks, propositional logic is also called sentential logic, statement logic, etc. They are certainly differences between propositions, sentences, and statements. However, those differences are not so important for our discussion here, and in most cases, these terms can be used interchangeably.

Basic Truth-Functional Connectives	Standard English Expression	Symbols
Conjunction	P and Q	P & Q
Negation	not P	~P
Disjunction	P or Q	P v Q
Material Conditional	if P, then Q	P→Q

Truth tables can be used to demonstrate the meaning of compound propositions such as "If p then Q" e.g., "If Mary is going to the party, then she will leave work early." Let's start with truth tables for the basic propositions:

Conjunction: p and q. For conjunction "p and q," e.g., "The car is red and very fast," both p and q must be true in order for the conjunction to be true, as the following table indicates:

Truth Table for the conjunction		
p	***q***	***p and q***
T	**T**	**T**
T	**F**	**F**
F	**T**	**F**
F	**F**	**F**

As you see, any combination of truth and falsity for p and q except p and q both being true leads to the conjunction being false.

Disjunction: p or q. In logic, the "or" disjunction is treated as an **inclusive** "or", i.e., for "p or q" to be true at least one of the propositions has to be true, but both can also be true. In ordinary English we often use the **exclusive** form of "or" as in "We should paint the room blue or white" (but clearly not both!). But we also use the inclusive form: e.g., to graduate the student must have two years instruction in either English or French." Clearly you would not be excluded for having both. For an inclusive disjunction, only one of the terms needs to be true, but both terms could be true, hence the first row as shown. If, however, an exclusive disjunction is involved—"p or q but not both"—then the first row, in which both p and q are true, would show the disjunction as false

Truth Table for the (<u>exclusive</u>) disjunction		
p	***q***	***p or q***
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

Truth Table for the (<u>inclusive</u>) disjunction		
p	***q***	***p or q***
T	**T**	**T**
T	**F**	**T**
F	**T**	**T**
F	**F**	**F**

Conditional: If p, then q. This one's a little tricky: if p is true, and q is true, the conditional is true—that's what it indicates, that if p is the case, then q is the case. And if p is true, and q is false, the conditional is false. If I said to you "If p, then q" and you experienced p, but q didn't happen, then what I said to you was obviously false. So much for the first two rows of the table.

Truth Table for the conditional		
p	***q***	***If p then q***
T	**T**	**T**
T	**F**	**F**
F	**T**	**T**
F	**F**	**T**

You'd be thinking quite reasonably that if you said that in the case of the next two rows, when p is false, the conditional, if p then q, is "not applicable" or "neither true nor false": the conditional starts with "If p is the case" so if we don't have p to start with, we can't go anywhere. But “neither true nor false” is not an option in propositional logic, so regardless of whether q is true or false, as long as p is false, the conditional is considered true. This might help: “If p, then q” is false only when p is true, but nevertheless q doesn’t follow (q is false)— which is the second row of the table, the only one marked false.

Now that we have explicated the meaning of the key terms in propositional logic, we turn to consider which argument forms using these terms are valid.

Let’s look at the following argument:

1. If the car runs, then it has fuel
2. It runs

Conclusion: It has fuel

This argument is obviously valid. If the premises are true, the conclusion must be true. Of course if it was an electric car then premise 1 would not be true, but the argument would still be valid.

This argument is a type of argument called **Modus Ponens** and is a very common type of argument form. It can be symbolized as shown in the following table.

Modus Ponens (MP) Example	**Argument Form**
If the car runs, then it has fuel.	If P, then Q
It runs.	P
Conclusion: It has fuel.	Q

Using our test for validity (if the premises are true, the conclusion must be true), we can see that arguments of the form **MP** such as the car example above are clearly valid. As mentioned, the car could be an electric car, but then premise 1 would not be true and the conclusion would be false. **Validity only guarantees a true conclusion if the premises are true.**

The next form of a valid argument is called **Modus Tollens**. It too is a very common type of argument form. Take this example which illustrates Modus Tollens, but also how the consequent can express a necessary condition.**Modus Tollens (MT) Argument Example**	**Argument Form**
If there is fire, then there is oxygen	If P, then Q
There is no oxygen	**Not Q**
Conclusion There will be no fire.	Not P

We can easily see why this argument is valid when we note that the second premise claims that a **necessary condition** for P is not present: No oxygen, then of course, no fire.

Let's look at the car and fuel example:

2A. Modus Tollens (MT) Example	**Argument Form**
If the car runs, then it has fuel.	If P, then Q

It has no fuel.	Not Q
Conclusion: It won't run	Not P

Let's look at a similar but ***invalid*** argument form: the fallacy of **Affirming the Consequent**. The *consequent* refers to the clause that follows the "then".

Fallacy of Affirming the Consequent Example	**Argument Form**
If the car runs, then it has fuel.	If P, then Q
It has fuel.	**Q**
Conclusion: It runs.	P

We can see that the argument is not valid. We all know of occasions in which a car has had fuel, but sadly would not run. The error here is to take the necessary condition for the car running, namely fuel, and treat it as if it were a sufficient condition. (See the explanation of necessary and sufficient in the text, Chapter 3, p. 65.)

We can use this way of looking at arguments to understand what was wrong with the objection which Nancy might have made to her Dad's position in the following argument:

1. If a policy is good for the economy, then it is good for the poor
2. Raising the minimum wage is good for the poor

Conclusion: Raising the minimum wage is good for the economy.

We have seen that this argument is not valid. If she had made this argument, Nancy would have committed the fallacy of affirming the consequent. In other words, in this conclusion Nancy treated “being good for the poor” as if it were a *sufficient condition* for “being good for the economy,” whereas her Dad was arguing the other way around, that *“being good for the economy*” was a *sufficient condition* for “being good for the poor”. Just a small twist of the argumentative order, and yet this makes the difference between a valid and invalid argument.

There is another argument form that looks quite similar to the valid argument forms we saw above but is not valid.

1. If you are a vegetarian, then you will be kind to animals
2. You are not a vegetarian

Conclusion: You will not be kind to animals

It looks similar to several of the arguments above but is not valid. It actually commits the **Fallacy of Denying the Antecedent**. “Antecedent” means that which preceded (your grandparents are some of your antecedents). In this case “antecedent” refers to the clause after the “if”.

Fallacy of Denying the Antecedent Example	**Argument Form**
If the car runs, then it has fuel.	If P, then Q
It won't run.	Not P
Conclusion: It has no fuel.	Not Q

Again, we all realize that the absence of fuel is one reason (a sufficient reason!) why a car might not run, but we cannot infer with certainty from the fact that the car will not run that it does not have fuel (e.g., the battery might be dead). And even if all vegetarians are kind to animals, the fact that someone is not a vegetarian is clearly not grounds for asserting that they are generally unkind to animals.

Click here for videos that introduce propositional logic using a formal symbolism. The production quality is not very professional, but they may be helpful.