PART I: TRUTH-FUNCTIONAL LOGIC

B. LOGICAL ASSESSMENT

4. In each case, determine whether the first schema implies the second. If implication fails to hold, exhibit a truth-assignment that witnesses this fact.

(a) \( p, r \equiv q, r \)
(b) \( p \equiv q \lor r \)
(c) \( p \lor r \equiv q \lor r \)
(d) \( \neg(p \lor q) \lor r, q \)
(e) \( (p \lor q) \cdot (p \supset r) \cdot (q \supset -r) \)
(f) \( p, -q \lor -p, r \)
(g) \( p \equiv -q \)

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6. One of the following statements truth-functionally implies the other, but not conversely. Determine which implication holds and verify that the converse implication does not.

(a) The police will act courageously and the pirates will be routed, if Fredric leads the attack; but if Fredric doesn’t lead the attack then the police won’t act courageously and the pirates won’t be routed.

(b) The pirates will be routed if and only if Fredric leads the attack and the police act courageously.

7. For each of the following arguments, determine whether the premises truth-functionally imply the conclusion:

(a) If Jones did not meet Smith last night, then either Smith was the murderer or Jones is lying.

If Smith wasn’t the murderer, then Jones did not meet Smith last night and the murder took place after midnight.

If the murder took place after midnight, then either Smith was the murderer or Jones is lying.

**Therefore, Smith was the murderer.**

(b) If Germany annexes Austria, then Czechoslovakia will be defensible only if France both honors her treaty obligations and arranges for the transport of troops across Poland or Rumania.

If Britain does not back the government in Prague, then Germany will annex Austria and France will fail to honor her treaty obligations.
THEREFORE, Czechoslovakia will be defensible only if Britain backs the government in Prague.

(c) If Deborah takes the job at State Street she will be overworked, and if she opts for the position at Fidelity she will be unappreciated.

If Deborah is either overworked or unappreciated, she will not be happy.

If Deborah works for Liberty Partners, she will be well-paid.

Of course, Deborah will take the job either at State Street, Fidelity, or Liberty Partners.

THEREFORE, if Deborah is happy, she is well-paid.

C. REFLECTION

3. Two schemata X and Y are incompatible iff their conjunction is unsatisfiable; they are contradictory iff their biconditional is unsatisfiable.

(a) Determine whether each of the following is true or false. If true, prove it; if false, provide a counterexample.

(i) If X and Y are incompatible, then they are also contradictory.

(ii) If X and Y are contradictory, then they are also incompatible.

(b) Show that if X and Y are contradictory, and X and Z are contradictory, then Y is equivalent to Z.
(c) Show that if \( X \) and \( Z \) are incompatible, and \( Y \) and the negation of \( Z \) are incompatible, then \( X \) and \( Y \) are incompatible.

6. Show that if a schema is constructed from sentence letters using only conjunction and disjunction then it is satisfiable but not valid.

PART II: MONADIC QUANTIFICATION THEORY

B. LOGICAL ASSESSMENT

8. In the arguments below, the premises monadically imply the conclusion. Show this, by paraphrasing and then using the method of §25.

(a) A fish is kosher if and only if it has fins and scales.
   No fish having scales lacks fins.
   Therefore, a fish is kosher provided it has scales.

(b) The chefs who catered the party are all from Manitoba.
   If no one from Manitoba is mentioned in the Times, then none of dominant wine traders are from Manitoba.
   Therefore, if any of the dominant wine traders catered the party, then some from Manitoba are mentioned in the Times.
No candidate who is either endorsed by labor or opposed by the press can carry the farm vote.

No one can be elected who does not carry the farm vote.

Any Democrat who is not endorsed by labor will be opposed by the press.

Therefore, no Democrat can be elected.

C. Reflection

2. Let $R_1, \ldots, R_n$ and $S$ be simple schemata, with $S$ universal. Show that $R_1, \ldots, R_n$ jointly imply $S$ iff the following holds: either $R_1, \ldots, R_n$ are jointly unsatisfiable or else the universal schemata among $R_1, \ldots, R_n$ jointly imply $S$.

PART III: POLYADIC QUANTIFICATION THEORY

A. Analysis

2. Symbolize, taking the universe of discourse to be the class of persons, and using “$S$” for “1 is a soprano”, “$T$” for “1 is a tenor”, “$L$” for “1 is louder than 2”, and “$R$” for “1 respects 2”:

(a) A soprano who respects all tenors fails to respect herself.
(b) A tenor who is louder than all sopranos is respected by all sopranos.
(c) No tenor who is louder than all sopranos respects any soprano.
(d) A tenor who is louder than some soprano is also louder than some tenor.
(e) There are sopranos who respect only those tenors who are louder than they.
(f) If a tenor respects all sopranos who respect him, then that tenor is respected by all sopranos.

3. With universe of discourse and vocabulary as in the previous problem, translate into clear, idiomatic English:
   (a) $\exists x(\exists y(T_x.R_{xy}.S_y) \supset (\exists y)[T_y.\ 
\quad (\forall x)(S_x \supset R_{xy})])$
   (b) $\exists x[T_x \cdot (\forall y)(S_y \cdot (\exists z)(T_z.R_{yz}) \supset R_{yx})]$
   (c) $\forall x[(\forall y)(S_y \supset R_{yx}) \supset (\forall y)(T_y \supset R_{yx})])$

8. Symbolize, taking the universe of discourse to be persons, and using “P” for “1 is a parent of 2”, “M” for “1 is male”, and “I” for “1 is identical to 2” (that is, “1 is the same person as 2”):
   (a) $x$ is $y$’s paternal grandfather.
   (b) $x$ and $y$ are sisters. (Caution: no one is her own sister!)
   (c) $x$ is a nephew of $y$.
   (d) $x$ is a half-sister of $y$.
   (e) $x$ is a first cousin of $y$. 

Exercises
B. LOGICAL ASSESSMENT

1. For each of the following schemata, find an interpretation with universe \{1, 2, 3, 4\} and nonempty extension of "F" that makes the schema true, and such an interpretation that makes the schema false:

(a) \((\exists x)(\forall y)(Fyx \supset Fyy)\)
(b) \((\forall x)(\forall y)(Fxy \supset (\exists z)(Fxz. Fyz))\)
(c) \((\forall x)[(\forall y)(Fyx \supset Fxy) \supset (\forall y)(Fxy \supset Fyx)]\)
(d) \((\exists x)(\exists y)(Fxy.Fyx) \cdot (\forall x)(\forall y)((\exists z)(Fxz.Fzy) \supset Fxy)\)

12. There is someone who admires anyone who overvalues all those they respect, but does not admire anyone who overvalues herself.
THEREFORE, if anyone overvalues all those they respect, then someone does not respect herself.

13. Everyone at the gala who had not befriended one of the donors had paid to attend.
Some Tories at the gala befriended only Tories.
No Tories had paid to attend.
THEREFORE, some Tories were donors.
C. REFLECTION

11. A schema is finitely satisfiable iff there exists an interpretation with finite universe of discourse that makes the schema true.

(a) Find a schema that is satisfiable but not finitely satisfiable.

(b) Show that there exists a search procedure for finite satisfiability.

(c) Show that there is no search procedure for schemata that are satisfiable but not finitely satisfiable.